Second-order estimates in anisotropic elliptic problems

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In recent years, various results showed that second-order regularity of solutions to the p-Laplace equation can be properly formulated in terms of the expression under the divergence, the so-called stress field.

I will discuss the extension of these results to the *anisotropic* p-Laplace problem, namely equations of the kind

$$-\text{div}\left(\mathcal{A}(\nabla u)\right) = f \quad \text{in } \Omega, \tag{1}$$

in which the stress field is given by $\mathcal{A}(\nabla u) = H^{p-1}(\nabla u)\, \nabla_\xi H(\nabla u)$, for a given norm $H = H(\xi)$ on $\mathbb{R}^n$ satisfying suitable ellipticity assumptions.

The $W^{1,2}$-Sobolev regularity of $\mathcal{A}(\nabla u)$ is established when f is square integrable, and both local and global estimates are obtained. The latter apply to solutions to homogeneous Dirichlet problems on either convex or sufficiently regular domains Ω.

A key point in our proof is an extension of *Reilly's identity* to the anisotropic setting.

This is based on joint works with A. Cianchi, G. Ciraolo, A. Farina and V.G. Maz'ya [1, 2].

References

[1] C.A. Antonini, G. Ciraolo, A. Farina, *Interior regularity results for inhomogeneous anisotropic quasilinear equations*, Math. Ann. Volume 387, pp. 1745-1776, (2023).

[2] C.A. Antonini, A. Cianchi, G. Ciraolo, A. Farina, V.G. Maz'ya, *Global second-order estimates in anisotropic elliptic problems*, arXiv preprint (2023) arXiv:2307.03052.